CS109 Section 1

Problem 1

A student is taking the computer-based GRE test. As the student answers questions, the test adapts its level of difficulty to their abilities. Let's say that the student is in the math section, and the test is trying to estimate whether or not they know geometry based on whether they answer the first geometry question correctly. The test's prior belief that the student knows geometry is q. The test also knows the following: Given that a student knows geometry, the probability that they solve the first question correctly is p_1. Additionally, given that a student does not know geometry, the probability that the solve the first questions correctly is p_2. Given that the student solves the first question correctly, what is the probability that they know geometry?

Problem 2

Continuing the story from Problem 1: Now the test is trying to estimate what level of experience a student has with geometry. There are 4 levels: Beginner, Intermediate, Expert, and Euclid, and each student is in exactly 1 level. The test's prior belief that a student is in a certain level is as follows: Beginner is b_1, Intermediate is b_2, Expert is b_3, and Euclid is b_4. The conditional probability that a student solves the first question correctly given their level is as follows: Beginner is d_1, Intermediate is d_2, Expert is d_3, and Euclid is d_4. Find the conditional probability that a student is in a certain level given that they solve the first question correctly. Do this for each of the 4 levels.

Problem 3

After a long night of programming, you have built a powerful, but slightly buggy, email spam filter. When you don't encounter the bug, the filter works very well, always marking a spam email as SPAM and always marking a non-spam email as GOOD. Unfortunately, your code contains a bug that is encountered 10% of the time when the filter is run on an email. When the bug is encountered, the filter always marks the email as GOOD. As a result, emails that are actually spam will be erroneously marked as GOOD when the bug is encountered. Let p denote the probability that an email is actually non-spam, and let q denote the conditional probability that an email is non-spam given that it is marked as GOOD by the filter. Determine q in terms of p.